Effects of unparticles on running of gauge couplings

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Abstract. Unparticles charged under a gauge group can contribute to the running of the gauge coupling. We show that a scalar unparticle of scaling dimension d contributes to the β function a term that is $(2-d)$ times that from a scalar particle in the same representation. This result has important implications for asymptotic freedom. An unparticle with $d > 2$, in contrast to its matter counterpart, can speed up the approach to asymptotic freedom for a non-Abelian gauge theory and has the tendency to make an Abelian theory also asymptotically free. For not spoiling the excellent agreement of the standard model (SM) with precision tests, the infrared cut-off, m, of such an unparticle would be high but might still be reachable at colliders such as LHC and ILC. Furthermore, if the unparticle scale $\Lambda_{\mathcal{U}}$ is high enough, unparticles could significantly modify the unification pattern of the SM gauge couplings. For instance, with three scalar unparticles of $d \sim 2.5$ in the adjoint representation of the strong gauge group but neutral under the electroweak one, the three gauge couplings would unify at a scale of $\sim 8 \times 10^{12}$ GeV, which is several orders of magnitude below the supersymmetric unification scale.

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Asymptotic freedom [1–3] had been historically stimulated by the suggestion [4] and observation [5] of Bjorken scaling in deeply inelastic scattering and contributed significantly to the establishment of quantum chromodynamics, a non-Abelian gauge theory, as the correct theory of strong interactions. It further offered insight in the endeavors [6] to unify strong and electroweak interactions: these interactions, though very different in their strength at low energies, may originate from a unified theory with a single coupling constant at some high energy scale [7], since the strength of strong interactions decreases as energy increases.

Central to the idea of asymptotic freedom is the negative sign of the β function, which determines renormalization group running of a coupling. It has been well established in quantum field theory [8] that only non-Abelian gauge theory can have a negative β if it does not contain too many matter fields. The latter restriction is necessary since all matter fields contribute positively to the β function. In particular, if there is only interaction with the matter fields in an Abelian gauge theory this is not asymptotically free.

Nevertheless, we want to show in this work that something very different from this can occur when gauge bosons are coupled to some effective degrees of freedom arising from a certain scale invariant sector at higher energies. Unparticles as suggested recently by Georgi [9] provide a con-

crete realization of such a degree of freedom, on which our work is based. We find that an unparticle charged under a gauge group contributes to the β function of the gauge coupling a term that is a multiple of the one from a matter field charged identically under the group. The multiplication factor can be of either sign, depending on the scaling dimension of the unparticle. This result has a major impact on asymptotic freedom. An unparticle with an appropriate scaling dimension can speed up the approach to asymptotic freedom of a non-Abelian theory and can make an Abelian theory become asymptotically free. If unparticles are charged under the SM gauge group, they will significantly modify the pattern of the grand unification.

Although our finding is surprising and in sharp contrast to the conventional statement on matter fields, we do not see any obvious conflict between the two. The reason is that the unparticle field is not a field in the conventional sense. Its quantum, an unparticle, is totally different from the particle quantum of a conventional field. It does not enjoy mass as one of its defining properties; instead, its kinematics is largely determined by the scaling dimension of its field, d, generally a non-integral number. The latter makes the unparticle field a non-local object that interacts differently from a conventional local field.

The original work of [9] has triggered intensive activities in unparticle physics in the past few months. Many of its salient features have been unveiled, investigated and applied [10–59] through interactions with the SM particles. Most of these interactions imply implicitly that unparticles

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are actually charged under the SM gauge group, $G_{\mathrm{SM}} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, but no attempt has been made to incorporate direct gauge interactions of unparticles until very recently in [52]. Although this issue has already been challenged in [9], it is non-trivial because of the non-local nature of the unparticle field. Fortunately, a similar problem was successfully treated some years ago in modeling low energy chiral dynamics of Goldstone bosons by dynamical quarks [60, 61]. A dynamical quark with a non-trivial momentum-dependent effective mass implies a non-local term in its Lagrangian that, when gauged, yields non-local interactions between the dynamical quark and gauge fields. The first phenomenological, interesting result has been obtained in [52]. We shall demonstrate that this non-local gauging of unparticle fields has far-reaching implications on the running properties of the gauge couplings themselves.

The scale invariance of the unparticle field determines the density of states in phase space of a scalar unparticle of momentum p to be proportional to $\theta(p^0)\theta(p^2)(p^2)^{d-2}$. Then unitarity considerations imply the following propagator [10, 11]:

$$\mathrm{i}D(p) = \frac{A_d}{2\sin(\pi d)} \frac{\mathrm{i}}{(-p^2 - \mathrm{i}\epsilon)^{2-d}}, \tag{1}$$

where A_d is a normalization factor inessential to our discussion. If scale invariance does not extend to arbitrarily low energy, some modifications to the above are required. The simplest way would be to add an infrared cut-off m^2 to the inside of the above power [21], and we shall follow this ansatz, although this will not affect our result in the ultraviolet.

The propagator is supposed to be derived from a Lagrangian quadratic in the unparticle field $\mathcal{U}(x)$. The latter is generally non-local for d non-integral or greater than 2, with the action

$$S_0 = \int \mathrm{d}^4 x\, \mathrm{d}^4 y\, \mathcal{U}^\dagger(x) \tilde{D}^{-1}(x-y)\mathcal{U}(y), \tag{2}$$

where $\tilde{D}^{-1}(z)$ is the Fourier transform of $D^{-1}(p)$. When $\mathcal{U}$ is charged under a local gauge group, the usual minimal coupling for a local field does not yield a gauge invariant term. Instead, a convenient way to build an invariant form is to invoke the Wilson line [52]:

$$S = \int \mathrm{d}^4 x\, \mathrm{d}^4 y\, \mathcal{U}^\dagger(x) \tilde{D}^{-1}(x-y)$$
$$\times P \exp\left[-\mathrm{i}g T^a \int_x^y A_\mu^a\, \mathrm{d}w^\mu\right] \mathcal{U}(y), \tag{3}$$

where P denotes path-ordering in the generators T^a of the gauge group in the unparticle representation. A systematic approach was developed in [60, 61] to derive interaction vertices from such an action.

For our purpose of computing the unparticle contribution to the β function, we consider the vacuum polarization diagrams shown in Fig. 1, which require the following two
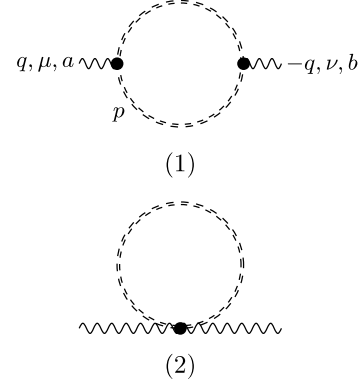

(1)

(2)

Fig. 1. Unparticles contributions to the vacuum polarization

vertices:

$$\mathrm{i}g\Gamma^{a\mu}(-(p+q), p; q)$$
$$= \mathrm{i}g T^a (2p+q)^\mu E_1(p; q),$$
$$\mathrm{i}g^2 \Gamma^{ab\mu\nu}(-(p+q_1+q_2), p; q_1, q_2)$$
$$= \mathrm{i}g^2 \left\{ \{T^a, T^b\} g^{\mu\nu} E_1(p; q_1+q_2) \right.$$
$$+ T^a T^b (2p+q_2)^\nu (2p+2q_2+q_1)^\mu E_2(p; q_2, q_1)$$
$$\left. + T^b T^a (2p+q_1)^\mu (2p+2q_1+q_2)^\nu E_2(p; q_1, q_2) \right\}, \tag{4}$$

where all momenta are meant to be incoming with the first two for the unparticles and others for the gauge bosons, and the recursive form factors are

$$E_0(p) = D^{-1}(p),$$
$$E_1(p; q_1) = \frac{E_0(p+q_1) - E_0(p)}{(p+q_1)^2 - p^2},$$
$$E_2(p; q_1, q_2) = \frac{E_1(p; q_1+q_2) - E_1(p; q_1)}{(p+q_1+q_2)^2 - (p+q_1)^2}. \tag{5}$$

The case of scalar particles is nicely recovered in the limit $d \to 1$ by noting that $\frac{A_d}{2\sin(\pi d)} \to -1$, $E_1 \to 1$ and that the recursion terminates at $E_2 \to 0$.

The imaginary part of the diagrams has been computed in [52] in a sophisticated manner. We shall do a complete calculation for the whole diagrams, and the result turns out to be surprisingly simple. The first diagram is

$$\mathrm{i}\mathcal{A}_1^{\mu\nu} = g^2 \mathrm{tr} T^a T^b \int \frac{\mathrm{d}^n p}{(2\pi)^n} \frac{(2p+q)^\mu (2p+q)^\nu}{(2pq+q^2)^2}$$
$$\times \left[\frac{D(p)}{D(p+q)} + \frac{D(p+q)}{D(p)} - 2 \right], \tag{6}$$

which is symmetric in $(2-d) \leftrightarrow (d-2)$ as can be seen by $p \to -(p+q)$. We work in n dimensions, where the power in $D(p)$ should be replaced by $\frac{n}{2} - d$ for consistency, but this does not affect the extraction of the β function. The second diagram is

$$\mathrm{i}\mathcal{A}_2^{\mu\nu} = -g^2 \mathrm{tr} T^a T^b \int \frac{\mathrm{d}^n p}{(2\pi)^n} D(p) \left\{ 2g^{\mu\nu} E_1(p; 0) \right.$$
$$+ (2p-q)^\nu (2p-q)^\mu E_2(p; -q, q)$$
$$\left. + (2p+q)^\mu (2p+q)^\nu E_2(p; q, -q) \right\}, \tag{7}$$

where

$$E_1(p; 0) = -\frac{2-d}{m^2 - p^2 - i\epsilon} \frac{1}{D(p)}. \qquad (8)$$

The second term in $i\mathcal{A}_2^{\mu\nu}$ can be made equal to the third one by $p \to -p$ and using $E_2(-p; -q, q) = E_2(p; q, -q)$. A little algebra gives

$$i\mathcal{A}_2^{\mu\nu} = -2g^2 \mathrm{tr} T^a T^b \int \frac{\mathrm{d}^n p}{(2\pi)^n}$$
$$\times \left\{ \left[\frac{(2p+q)^\mu (2p+q)^\nu}{2qp+q^2} - g^{\mu\nu} \right] \frac{2-d}{m^2 - p^2 - i\epsilon} \right.$$
$$\left. + \frac{(2p+q)^\mu (2p+q)^\nu}{(2qp+q^2)^2} \left[\frac{D(p)}{D(p+q)} - 1 \right] \right\}, \qquad (9)$$

where the first term is odd in $(2-d)$. By $p \to -(p+q)$, the second one can be cast in a form that amounts to $(2-d) \to (d-2)$, and averaging it with the original form yields exactly $-i\mathcal{A}_1^{\mu\nu}$. Thus we have

$$i(\mathcal{A}_1 + \mathcal{A}_2)^{\mu\nu} = -2g^2 \mathrm{tr} T^a T^b \int \frac{\mathrm{d}^n p}{(2\pi)^n} \frac{2-d}{m^2 - p^2 - i\epsilon}$$
$$\times \left[\frac{(2p+q)^\mu (2p+q)^\nu}{2qp+q^2} - g^{\mu\nu} \right]. \qquad (10)$$

Applying again the same trick to the first term in the above sum yields the final answer:

$$i(\mathcal{A}_1 + \mathcal{A}_2)^{\mu\nu}$$
$$= (2-d)g^2 \mathrm{tr} T^a T^b \left[\int \frac{\mathrm{d}^n p}{(2\pi)^n} \frac{2g^{\mu\nu}}{m^2 - p^2 - i\epsilon} \right.$$
$$\left. + \int \frac{\mathrm{d}^n p}{(2\pi)^n} \frac{(2p+q)^\mu (2p+q)^\nu}{[m^2 - (p+q)^2 - i\epsilon][m^2 - p^2 - i\epsilon]} \right]. \qquad (11)$$

Namely, the scalar unparticle contribution to the vacuum polarization is $(2-d)$ times that of the scalar particle in the same representation.

A few comments are in order. The above manipulation is much simpler than that in [52] while yielding an even stronger result: the relation between the unparticle and particle contributions holds not just for the imaginary part as explicitly shown there, but for the whole amplitude. This result is consistent with the 'slick' argument in that paper based on path integrals, which however could be dangerous due to the presence of ultraviolet singularities and non-locality. The relation for the imaginary part and the conventional optical theorem for particles were further employed in [52] to obtain the cross section for pair production of colored unparticles via one gluon exchange from the initial quark and anti-quark state. A potential problem with the naive application of the theorem in the present model is discussed in the appendix. The conclusion drawn from the discussion is that the subtlety does not pose an obstacle to our main interest of calculating unparticle effects on the running of gauge couplings.

Having obtained the vacuum polarization, the contribution to the β function can be written down directly for $n_\mathcal{U}$ the species of unparticles in the representation $r_\mathcal{U}$ of the gauge group:

$$\beta(g)_\mathcal{U} = (2-d)\frac{1}{4} n_\mathcal{U} \frac{g^3}{(4\pi)^2} \frac{4}{3} C(r_\mathcal{U}), \qquad (12)$$

where $(2-d)$ is as computed, $\frac{1}{4}$ for scalars and the last factors are standard for a fermion particle multiplet in the representation $r_\mathcal{U}$ with $\mathrm{tr} T^a T^b = C(r_\mathcal{U})\delta^{ab}$. This result is simple and surprising. As argued in [9], an unparticle with scale dimension d kinematically looks like a number d of invisible massless particles, but its contribution to the β function does not look like a matter field but something opposite: the d term has a minus sign. That $\beta(g)_\mathcal{U}$ happens to vanish at $d = 2$ is a result in four dimensions; at this value all interactions are absent as is clear from the action. Concerning d, there are no real constraints but the one from analysis of unitary representations in conformal theory: $d \geq 1$ for a scalar object [62]. Thus, the unparticle term could be of either sign.

Particularly interesting is the possibility that $d > 2$. In this case, unparticles behave oppositely to matter fields in affecting the running of a gauge coupling. A non-Abelian gauge theory can approach the asymptotic freedom faster when it is coupled to such unparticles fields. An Abelian gauge theory, which is otherwise not asymptotically free, could become asymptotically free if the unparticle contribution dominates over that of matter fields. At first sight, this might look like of merely academic interest since the running of the SM gauge couplings has been well tested; see for instance [63] for a recent review on experimental tests of asymptotic freedom in QCD. This is not necessarily true. Unparticles as a remnant of some scale invariant theory at high energy could become relevant at a scale that is a bit higher than a few hundred GeV, up to which the running of the SM gauge couplings has only been measured. To avoid spoiling the well-tested region, we thus require an infrared cut-off m for unparticles that is high enough. Supposing that the unparticle scale $\Lambda_\mathcal{U}$ is much higher than the electroweak scale, we shall thus investigate how the unification of the gauge couplings in SM could be affected at high energy scales in between.

The β functions for the three gauge couplings in SM are

$$\beta(g_s)_{\mathrm{SM}} = -\frac{g_s^3}{4\pi^2} \frac{7}{4}, \quad \beta(g)_{\mathrm{SM}} = -\frac{g^3}{4\pi^2} \frac{5}{6},$$
$$\beta(g')_{\mathrm{SM}} = \frac{g'^3}{4\pi^2} \frac{5}{3}. \qquad (13)$$

The unparticle terms should be added to the above accordingly when they are charged under a gauge group. At the scale M, those couplings are expected to unify [6,7]:

$$g_s^2(M) = g^2(M) = \frac{5}{3} g'^2(M). \qquad (14)$$

Their values at the Z boson mass have been well determined. For those, we use the numbers from the Particle Data Group: $g_s^{-2}(m_Z) = 0.654$, $g^{-2}(m_Z) = 2.354$, $g'^{-2}(m_Z) = 7.826$ with $m_Z = 91.19\,\mathrm{GeV}$. For illustration purposes, we consider a simple scenario that contains

$n_{\mathcal{U}}$ species of scalar unparticles with dimension d, all in the same representation $r_{\mathcal{U}}$ of $SU(3)_c$, but neutral under $SU(2)_L \times U(1)_Y$. In other words, g^{-2} and $\frac{5}{3}g'^{-2}$ meet at the same scale as they would in SM, $M \sim 8 \times 10^{12}$ GeV, where $g^{-2}(M) = 3.418$.

The value of $g_s^{-2}(M)$ at the scale M is shown in Fig. 2 as a function of d for $n_{\mathcal{U}}$ species of scalar unparticles in the fundamental (3) or adjoint (G) representation of $SU(3)_c$. The horizontal dash line corresponds to the point where all three gauge couplings unify with the help of these unparticles. It is interesting that with a few simply arranged unparticles the unification scale for the SM gauge couplings could be several orders of magnitude lower than the one in the supersymmetric standard model [64]. Unparticle physics may provide an alternative to ordinary new physics for grand unification.

Unparticles should interact with particles observed in experiments to be physically relevant. It is natural that they carry charges under the SM gauge group. Due to non-trivial scale dimensions of unparticle fields, their gauge theory generally is non-local. While the interactions in such a theory may be complicated, some properties are reachable without ambiguities. Although these interactions may back-react to break scale invariance of unparticles below the electroweak scale or due to renormalization effects, their leading effects to the running of gauge couplings from the electroweak scale to the unparticle scale can be consistently studied. We have shown that an unparticle field contributes to the β function of a gauge coupling in a simple manner: the ordinary result for a matter field multiplied by a factor depending on the unparticle's scale dimension. The point is that this factor could be of either sign, instead of being positive-definite as in the matter case. An Abelian gauge theory could become asymptotically free if the gauge boson interacts with an unparticle of a large enough scale dimension. This new feature in the β function could have implications in the other context of gauge theory.

The effect on the β function could potentially modify the pattern of unification of gauge couplings. We have illustrated this with a simple scenario showing that the unification scale could be made several orders of magnitude lower than the one in supersymmetric unification. But there are many uncertain factors in this unparticle-assisted unification. It depends on the detailed arrangement of unparticles under the SM gauge group, to which there currently seems to be no guide. More uncertain is perhaps the impact from high energy physics that produces unparticles; in our illustrative example, we have implicitly assumed that the unification scale is lower than the scale at which unparticle degrees of freedom appear.

There are several things worthy to be explored. Unparticle effects could be avoided by setting a large enough cut-off m. But unparticles will become more relevant and interesting if it is not too high. For instance, it might be possible that some effects are observable at high energy colliders such as the LHC or ILC without spoiling the precision data at lower energies. They could also be detected in ultra-high energy processes in astrophysics. All of this will depend on a more or less complete model for unparticles in the framework of electroweak and strong interactions.

Appendix

We show by symmetry analysis that the optical theorem for particle scattering very likely breaks down for unparticles in the present model when it is straightforwardly applied. We take the same process as computed in [52], i.e., $q(p_1)\bar{q}(p_2) \to \mathcal{U}(k_1)\bar{\mathcal{U}}(k_2)$, where quarks are treated massless and the unparticle $\mathcal{U}$ has the color representation $r_{\mathcal{U}}$ with the generators T^a normalized as $\mathrm{tr}T^aT^b = C(r_{\mathcal{U}})\delta^{ab}$. The amplitude is

$$i\mathcal{A} = \bar{v}(p_2)ig_s\gamma_\mu\frac{\lambda^a}{2}u(p_1)\frac{-i}{(p_1+p_2)^2}ig_sT^a\frac{(k_1-k_2)^\mu}{k_1^2-k_2^2}$$
$$\times\left[\frac{1}{D(k_1)} - \frac{1}{D(k_2)}\right]. \tag{A.1}$$

Note that $D(k_i)$ is complex for $K_i = k_i^2 - m^2 > 0$:

$$D(k_i) = \frac{A_d}{2\sin(d\pi)}K_i^{d-2}e^{-i(\pi-\epsilon)d}. \tag{A.2}$$

Doing color and spin summation and averaging and attaching the phase space factors for unparticles, the differential cross section is

$$d\sigma = \sin^2(d\pi)\left(N_c^2 - 1\right)C(r_{\mathcal{U}})\frac{g_s^4}{9s^3}$$
$$\times\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}(2\pi)^4\delta^4(p_1+p_2-k_1-k_2)$$
$$\times\left[2p_1(k_1-k_2)p_2(k_1-k_2) - p_1p_2(k_1-k_2)^2\right]$$
$$\times\frac{1}{(K_1-K_2)^2}\left[\left(\frac{K_1}{K_2}\right)^{2-d} + \left(\frac{K_2}{K_1}\right)^{2-d} - 2\right],$$
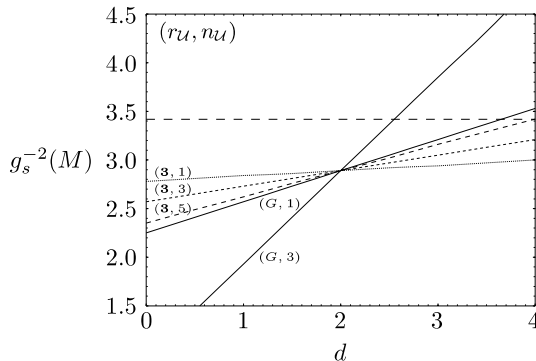$$\tag{A.3}$$

Fig. 2. $g_s^{-2}(M)$ as a function of d for $n_{\mathcal{U}}$ species of scalar unparticles in the fundamental or adjoint representation of $SU(3)_c$

where the step functions for k_i are implied and $N_c = 3$. The apparent singularity at $k_1^2 = k_2^2$ can be removed using a trick [52]:

$$\left(\frac{K_1}{K_2}\right)^{2-d} = K_1 \frac{1}{K_1^{d-1} K_2^{d-2}}$$
$$= \frac{K_1}{\Gamma(d-1)\Gamma(2-d)} \int_0^1 dx \frac{x^{1-d}(1-x)^{d-2}}{K_1(1-x) + K_2 x},$$

(A.4)

so that all K_i factors combine to

$$\frac{1}{\Gamma(d-1)\Gamma(2-d)}$$
$$\times \int_0^1 dx \frac{x^{2-d}(1-x)^{d-2}(2x-1)}{[K_1(1-x) + K_2 x][K_2(1-x) + K_1 x]}.$$

(A.5)

For the symmetry analysis, we keep only relevant factors:

$$d\sigma \propto \sin^3(d\pi)$$
$$\times \int_0^1 dx \frac{x^{2-d}(1-x)^{d-2}(2x-1)}{[K_1(1-x) + K_2 x][K_2(1-x) + K_1 x]}.$$

(A.6)

The pre-factor changes sign when $(2-d) \to (d-2)$, and so does the integral as can be seen by $x \to (1-x)$. The cross section thus obtained is an even function of $(2-d)$, in contrast to the vacuum polarization, which is odd. Its first non-vanishing derivative at $d = 2$ appears in the fourth order, and thus the cross section is not a linear function of d in the neighborhood of $d = 2$. This would be sufficient to signal the breakdown of the particle optical theorem in the unparticle gauge model.

This by itself should not be too surprising. Unparticles do not correspond to asymptotic states in the usual sense of the word, for which the issue of how to define an S matrix is not yet settled. The calculations in the literature on processes involving unparticles in the initial or final state seem to go through the check of the naive optical theorem, as the author has checked, but all those are restricted to unparticles that are not charged under any gauge group. The new element in the gauge model of [52] is the presence of the propagator $D(p)$ in the interaction vertices. For non-integral d, this modifies the analyticity properties in a non-trivial way. For instance, the interaction vertices are even not Hermitian when an involved unparticle is above its infrared cut-off, $p^2 > m^2$.

The situation is similar to that in the non-local chiral quark model. One of the purposes to introduce a dynamical quark mass had been to mimic the QCD effects on the chiral dynamics of Goldstone bosons and to compute the low energy constants in the chiral Lagrangian in particular [65]. A form for dynamical quark mass is generally assumed for Euclidean momentum, whose limit in the deep Euclidean region is well motivated. This is sufficient for dynamics of Goldstone bosons, and no problem is anticipated when dynamical quarks are confined in loops. This

is also the case here: the unparticle is treated as an interpolating field or effective degree of freedom from certain scale invariant physics whose loop effects on dynamics of gauge fields are studied. No problem is thus expected for the β function calculated in this paper that is related to the real part of the vacuum polarization due to virtual unparticles. When trying to compute quantities of a 'free' quark, however, e.g., the axial vector coupling of the constituent quark [66], care must be taken and additional assumptions are required for a dynamical quark mass in the time-like region.

The possible breakdown of the optical theorem due to the change of analyticity properties has a well-known analog in non-commutative field theory. When Feynman rules are naively derived, the optical theorem is indeed broken when time does not commute with space [67]. The breakdown can be attributed to the appearance of phase factors that involve the zero component of momentum in the naive Feynman rules and modify the analyticity properties of the Green functions in a significant way [68,69]. A careful treatment with them results in Feynman rules fulfilling the optical theorem.

References

1. G. 't Hooft, unpublished
2. D.J. Gross, F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973)
3. H.D. Politzer, Phys. Rev. Lett. **30**, 1346 (1973)
4. J.D. Bjorken, Phys. Rev. **179**, 1547 (1969)
5. E.D. Bloom et al., Phys. Rev. Lett. **23**, 930 (1969)
6. H. Georgi, S.L. Glashow, Phys. Rev. Lett. **32**, 438 (1974)
7. H. Georgi, H.R. Quinn, S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974)
8. S.R. Coleman, D.J. Gross, Phys. Rev. Lett. **31**, 851 (1973)
9. H. Georgi, Phys. Rev. Lett. **98**, 221601 (2007) [hep-ph/0703260]
10. H. Georgi, Phys. Lett. B **650**, 275 (2007) [0704.2457 [hep-ph]]
11. K. Cheung, W.Y. Keung, T.C. Yuan, Phys. Rev. Lett. **99**, 051803 (2007) [0704.2588 [hep-ph]]
12. M. Luo, G. Zhu, Phys. Lett. B **659**, 341 (2008) [0704.3532 [hep-ph]]
13. C.H. Chen, C.Q. Geng, Phys. Rev. D **76**, 115003 (2007) [0705.0689 [hep-ph]]
14. G.J. Ding, M.L. Yan, Phys. Rev. D **76**, 075005 (2007) [0705.0794 [hep-ph]]
15. Y. Liao, Phys. Rev. D **76**, 056006 (2007) [0705.0837 [hep-ph]]
16. T.M. Aliev, A.S. Cornell, N. Gaur, Phys. Lett. B **657**, 77 (2007) [0705.1326 [hep-ph]]
17. S. Catterall, F. Sannino, Phys. Rev. D **76**, 034504 (2007) [0705.1664 [hep-lat]]
18. X.Q. Li, Z.T. Wei, Phys. Lett. B **651**, 380 (2007) [0705.1821 [hep-ph]]
19. C.D. Lu, W. Wang, Y.M. Wang, Phys. Rev. D **76**, 077701 (2007) [0705.2909 [hep-ph]]
20. M.A. Stephanov, Phys. Rev. D **76**, 035008 (2007) [0705.3049 [hep-ph]]
21. P.J. Fox, A. Rajaraman, Y. Shirman, Phys. Rev. D **76**, 075004 (2007) [0705.3092 [hep-ph]]

22. N. Greiner, Phys. Lett. B **653**, 75 (2007) [0705.3518 [hep-ph]]
23. H. Davoudiasl, Phys. Rev. Lett. **99**, 141 301 (2007) [0705.3636 [hep-ph]]
24. D. Choudhury, D.K. Ghosh, Mamta, Phys. Lett. B **658**, 148 (2008) [0705.3637 [hep-ph]]
25. S.L. Chen, X.G. He, Phys. Rev. D **76**, 091 702 (2007) [0705.3946 [hep-ph]]
26. T.M. Aliev, A.S. Cornell, N. Gaur, JHEP **0707**, 072 (2007) [0705.4542 [hep-ph]]
27. P. Mathews, V. Ravindran, Phys. Lett. B **657**, 198 (2007) [0705.4599 [hep-ph]]
28. S. Zhou, Phys. Lett. B **659**, 336 (2008) [0706.0302 [hep-ph]]
29. G.J. Ding, M.L. Yan, 0706.0325 [hep-ph]
30. C.H. Chen, C.Q. Geng, Phys. Rev. D **76**, 036 007 (2007) [0706.0850 [hep-ph]]
31. Y. Liao, J.Y. Liu, Phys. Rev. Lett. **99**, 191 804 (2007) [0706.1284 [hep-ph]]
32. M. Bander, J.L. Feng, A. Rajaraman, Y. Shirman, Phys. Rev. D **76**, 115 002 (2007) [0706.2677 [hep-ph]]
33. T.G. Rizzo, JHEP **0710**, 044 (2007) [0706.3025 [hep-ph]]
34. K. Cheung, W.Y. Keung, T.C. Yuan, Phys. Rev. D **76**, 055 003 (2007) [0706.3155 [hep-ph]]
35. H. Goldberg, P. Nath, 0706.3898 [hep-ph]
36. S.L. Chen, X.G. He, H.C. Tsai, JHEP **0711**, 010 (2007) [0707.0187 [hep-ph]]
37. R. Zwicky, Phys. Rev. D **77**, 036 004 (2008) [0707.0677 [hep-ph]]
38. T. Kikuchi, N. Okada, Phys. Lett. B **661**, 360 (2008) [0707.0893 [hep-ph]]
39. R. Mohanta, A.K. Giri, Phys. Rev. D **76**, 075 015 (2007) [0707.1234 [hep-ph]]
40. C.S. Huang, X.H. Wu, 0707.1268 [hep-ph]
41. N.V. Krasnikov, Int. J. Mod. Phys. A **22**, 5117 (2007) [0707.1419 [hep-ph]]
42. A. Lenz, Phys. Rev. D **76**, 065 006 (2007) [0707.1535 [hep-ph]]
43. J.J. van der Bij, S. Dilcher, Phys. Lett. B **655**, 183 (2007) [0707.1817 [hep-ph]]
44. D. Choudhury, D.K. Ghosh, 0707.2074 [hep-ph]
45. H. Zhang, C.S. Li, Z. Li, Phys. Rev. D **76**, 116 003 (2007) [0707.2132 [hep-ph]]
46. X.Q. Li, Y. Liu, Z.T. Wei, 0707.2285 [hep-ph]
47. Y. Nakayama, Phys. Rev. D **76**, 105 009 (2007) [0707.2451 [hep-ph]]
48. N.G. Deshpande, X.G. He, J. Jiang, Phys. Lett. B **656**, 91 (2007) [0707.2959 [hep-ph]]
49. T.A. Ryttov, F. Sannino, Phys. Rev. D **76**, 105 004 (2007) [0707.3166 [hep-th]]
50. R. Mohanta, A.K. Giri, Phys. Rev. D **76**, 057 701 (2007) [0707.3308 [hep-ph]]
51. A. Delgado, J.R. Espinosa, M. Quiros, JHEP **0710**, 094 (2007) [0707.4309 [hep-ph]]
52. G. Cacciapaglia, G. Marandella, J. Terning, JHEP **0801**, 070 (2008) [0708.0005 [hep-ph]]
53. M. Neubert, Phys. Lett. B **660**, 592 (2008) [0708.0036 [hep-ph]]
54. M.X. Luo, W. Wu, G.H. Zhu, Phys. Lett. B **659**, 349 (2008) [0708.0671 [hep-ph]]
55. S. Hannestad, G. Raffelt, Y.Y.Y. Wong, Phys. Rev. D **76**, 121 701 (2007) [0708.1404 [hep-ph]]
56. K.J.Y. Hamada, A. Minamizaki, A. Sugamoto, Mod. Phys. Lett. A **23**, 237 (2008) [0708.2127 [hep-ph]]
57. N.G. Deshpande, S.D.H. Hsu, J. Jiang, Phys. Lett. B **659**, 888 (2008) [0708.2735 [hep-ph]]
58. P.K. Das, Phys. Rev. D **76**, 123 012 (2007) [0708.2812 [hep-ph]]
59. G. Bhattacharyya, D. Choudhury, D.K. Ghosh, Phys. Lett. B **655**, 261 (2007) [0708.2835 [hep-ph]]
60. J. Terning, Phys. Rev. D **44**, 887 (1991)
61. B. Holdom, Phys. Rev. D **45**, 2534 (1992)
62. G. Mack, Commun. Math. Phys. **55**, 1 (1977)
63. S. Bethke, Prog. Part. Nucl. Phys. **58**, 351 (2007) [hep-ex/0606035]
64. P. Langacker, M.X. Luo, Phys. Rev. D **44**, 817 (1991)
65. B. Holdom, J. Terning, K. Verbeek, Phys. Lett. B **245**, 612 (1990)
66. X.Y. Li, Y. Liao, Phys. Lett. B **318**, 537 (1993)
67. J. Gomis, T. Mehen, Nucl. Phys. B **591**, 265 (2000) [hep-th/0005129]
68. Y. Liao, K. Sibold, Eur. Phys. J. C **25**, 469 (2002) [hep-th/0205269]
69. Y. Liao, K. Sibold, Eur. Phys. J. C **25**, 479 (2002) [hep-th/0206011]